MHD Free Convective Dissipative Flow with Heat Source and Chemical Reaction.

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Abstract
The problem of heat and mass transfer of MHD free convective dissipative flow with heat absorption and chemical reaction effects are examined in this study. We assumed a constant suction velocity normal in the direction of the plate and under the Boussinesq approximation, the dimensional governing equations were formulated. The coupled governing equations were solved analytically by assuming a set of solutions for velocity, temperature and specie concentration and the graphical results are presented and discussed. We observed that velocity increases with increase in the heat absorption, porosity, chemical reaction and Grashof number but decreases with increase in the magnetic field. Furthermore, increase in heat source increase temperature but decreases with increase in oscillation parameter. While there is an increase in concentration due to increase in Soret, concentration decreases with increase in chemical reaction.

Keywords: Free convection, chemical reaction, suction velocity, magnetohydrodynamics(MHD), heat source

1.0 Introduction

In many processes, convective heat and mass transfer with chemical reaction are very important and these have attracted attention. For example, in processes like evaporation at the surface of a water body, drying, energy transfer in a wet cooling tower and the flow in a desert cooler. Related applications of this type of flow are found in many industries such as in the power industry where electrical energy is extracted directly from a moving conducting fluid which generates electric power, also in the spreading of chemical pollutants in plants as well as diffusion of medicine in blood veins. Chambre and Young (1958) have investigated a first chemical reaction paste a horizontal plate in a laminar flow. Dekha et al. (1994) examined the impact of the first homogeneous chemical reaction on the process of an unsteady flow past a vertical plate with a steady heat and mass transfer. Sarada and Shanker (2013) studied the effects of chemical reaction on an unsteady MHD flow past an infinite vertical porous plate with variable suction and heat convective mass transfer. Vyas et al (2012) studied the impact of dissipation on heat and mass transfer as a result of the upward vertical movement of the plate in a porous medium using perturbation method. Acharya et al (1999) investigated on heat and mass transfer over an accelerating surface with heat source in the presence of suction and injection. The work of Umamaheswar et al., (2016) investigated unsteady MHD free convection, warmth and mass exchange flow of a Newtonian flow past boundless vertical plate with homogeneous chemical reaction and heat absorption. Choudhary et al., (2016) presented

a numerical solution to an unsteady MHD flow and heat transfer over an expanding permeable surface with suction using fourth order Rugge Kutta method with shooting method.
Rao et al (2012) studied the effects of chemical reaction on an unsteady magnetohydrodynamics free convection fluid flow past a semi-infinite vertical plate implanted in a permeable medium with heat absorption. Makinde (2009) investigated the MHD boundary layer flow and mass transfer past a vertical plate in a porous medium with constant heat flux. Free convection flow in a porous medium has been investigated on because of its technological impart. Pop and Soundalgekar (1962) studied unsteady free convection flow past an infinite plate with constant suction and heat source. Chandrakala (2010) examined the free convection flow of a viscous incompressible fluid with uniform heat flux in the presence of thermal radiation. Hossain (1990) studied the effects of ohmic heating on the MHD free convection heat transfer for a Newtonian fluid. Soundalgekar (1979) presented the effects of mass transfer and free convection currents on the flow past an impulsively started vertical plate. This work is aimed at extending the work of Amos et al. (2018) to examine chemical reaction and heat source effects.

2.0 Mathematical Formation

Consider an unsteady free convective boundary layer flow of an incomprehensible, viscous, electrically conducting and chemically reactive fluid past a semi-infinite plate subject to uniform magnetic field and temperature dependent heat source. The x-axis is long along the plate while the y-axis is normal to it. The velocity components are u' and v' in the x' and y' directions respectively. The magnetic field is of strength $\bar{B}_0$ applied normal to the plate (wherein the magnetic Reynolds number is assumed small and hence the induced magnetic field is negligible). The plate temperature is T_w and the concentrationC_w, where $T_w > T_\infty, C_w > C_\infty$ and T_∞ and C_∞ are ambient temperature and concentration respectively. With Boussinesq approximation, the governing equations under above physical considerations are:

$$\frac{\partial v'}{\partial y'} = 0 \qquad (1)$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \upsilon\frac{\partial^2 u'}{\partial y'^2} + g\beta_T(\mathrm{T}' - \mathrm{T}'_\infty) + \mathrm{g}\,\beta_C(\mathrm{C}' - \mathrm{C}'_\infty) - \sigma \mathrm{B}_0^2 u' - \frac{\upsilon}{k'}u' \qquad (2)$$

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{Q_0}{\rho C_p}(\mathrm{T}' - \mathrm{T}'_\infty) \qquad (3)$$

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - Kr'(\mathrm{C}' - \mathrm{C}'_\infty) + D_1\frac{\partial^2 T'}{\partial y'^2} \qquad (4)$$

with boundary conditions:

$$\mathrm{u}' = 0, \quad \mathrm{T}' = \mathrm{T}'_w, \quad \mathrm{C}' = C'_w, \quad \mathrm{y}' = 0 \qquad (5)$$

$$\mathrm{u}' \to 0, \quad \mathrm{T}' \to \mathrm{T}'_\infty, \quad \mathrm{C}' \to C'_\infty, \text{ as } \quad \mathrm{y}' \to \infty \qquad (6)$$

where σ is electrical conductivity, β_T is the volumetric coefficient of thermal expansion, β_C is volumetric coefficient of expansion with concentration, g is acceleration due to gravity, C_p is specific heat at constant pressure, Q_0 is heat generation, ρ is density of the fluid, κ is thermal diffusivity, D is the specific diffusion coefficient, υ is the kinematic viscosity.
We assume constant suction velocity v' normal to the plate where v' is the scale of the suction such that
v' = *-1*. The negative plate indicates that the suction is toward the plate.

We now apply the following non-dimensional quantities:

$$y=\frac{v_0'y'}{\upsilon};\quad u=\frac{u}{U_0'};\quad v=\frac{v'}{v_0'};\quad t=\frac{v_0'^2}{\upsilon};\quad \theta=\frac{T'-T_\infty'}{T_w'-T_\infty'};\quad Kr'=\frac{Kr\upsilon}{v_0'^2};\qquad (7)$$

$$C=\frac{C'-C_\infty'}{C_w'-C_\infty'};\quad Gr=\frac{\upsilon gB_T(T_w'-T_\infty')}{U_0v_0'^2};\quad Gm=\frac{\upsilon gB_C(C_w'-C_\infty')}{U_0v_0'^2};\qquad (8)$$

$$Sc=\frac{\upsilon}{D};\quad \varphi=\frac{Q_0\upsilon}{\rho C_p v_0'^2};\quad M=\frac{\sigma B_0^2\upsilon}{\rho v_0'^2};\quad So=\frac{D_1}{\upsilon}\frac{(T_w'-T_\infty')}{(C_w'-C_\infty')};\quad k=\frac{k'v_0'^2}{\upsilon^2}\qquad (9)$$

Consequently, using (7), (8) and (9), equations (1) - (6) becomes;

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+Gr\theta+GmC-Nu\qquad (10)$$

$$\frac{\partial \theta}{\partial t}-\frac{\partial \theta}{\partial y}=\frac{\partial^2 \theta}{\partial y^2}+\varphi\theta\qquad (11)$$

$$Sc\frac{\partial C}{\partial t}-Sc\frac{\partial C}{\partial y}=\frac{\partial^2 C}{\partial y^2}-ScKrC+ScSo\frac{\partial^2 \theta}{\partial y^2}\qquad (12)$$

With boundary conditions:

$$u=0,\quad \theta=1,\quad C=1\quad at\quad y=0\qquad (13)$$

$$u\to 0,\quad \theta\to 0,\quad C\to 0\quad as\quad y\to\infty\qquad (14)$$

where

$N=M+\frac{1}{K}$, M is the magnetic field, Gr is Grashof number, Gm is the modified Grashof number, φ is the heat generation term, K is the porosity term, Sc is the Schmidt number and So is the Soret term.

3.0 Method of Solutions

In order to solve to equation (10) – (14), we assume a trial solution of the form

$$U(Y,t)=U_0(Y)e^{i\omega t};\quad \theta(Y,t)=\theta_0(Y)e^{i\omega t};\quad C(Y,t)=C_0e^{i\omega t};\qquad (15)$$

where ω is the frequency of oscillation.

We obtain the following

$$U_0''+U_0'-(i\omega+N)U_0=-Gr\theta_0-GmC_0\qquad (16)$$

$$\theta_0''+\theta'-(i\omega-\varphi)\theta_0=0\qquad (17)$$

$$C_0''+ScC_0'-Sc(i\omega+kr)C_0=-So\theta_0''\qquad (18)$$

With boundary conditions

$$U_0\to 0;\quad \theta_0\to 0;\quad C_0\to 0\quad \text{at}\quad y\to\infty\qquad (19)$$

$$U_0=0;\quad \theta_0=1;\quad C_0=1\quad \text{at}\quad y=0\qquad (20)$$

Solving equations (16) – (18) with the boundary conditions (19) – (20), and noting (15) we obtain

$$U=(\phi_2e^{m_1y}+\phi_3e^{m_2y}+\gamma_3e^{m_3y})e^{i\omega t}\qquad (21)$$

$$\theta=(e^{m_1y})e^{i\omega t}\qquad (22)$$

$$C=(\gamma_2e^{m_2y}+\phi_1e^{m_1y})e^{i\omega t}\qquad (23)$$

where $N=M+\frac{1}{K}$

$$m_1=-(\frac{1+\sqrt{1+4(i\omega-\varphi)}}{2};\qquad \phi_1=\frac{-m_1^2ScSo}{m_1^2+m_1Sc-Sc(i\omega+Kr)};\quad \gamma_2=1-\phi_1$$

$m_2 = -(\frac{Sc + \sqrt{Sc^2 + 4Sc(i\omega + Kr)}}{2})$; $\gamma_3 = -(\phi_2 + \phi_3)$; $\phi_2 = \frac{-(Gr\,\gamma_1 + Gm\phi_1)}{m_1^2 + m_1 - (i\omega + N)}$

$\phi_3 = \frac{-Gr\,\gamma_2}{m_2^2 + m_2 - (i\omega + N)}$; $m_3 = -(\frac{1 + \sqrt{1 + 4(i\omega + N)}}{2})$

Nuselts Number

$$Nu = -x\left(\frac{\partial T'}{\partial y}\right)_{y=0} \Rightarrow Nu\,\mathrm{Re}_x^{-1} = \left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \left(\frac{\partial \theta_0}{\partial y}\right)_{y=0} = m_1 \tag{24}$$

The Skin friction.

$$C_f = \frac{\tau_w}{\rho U_0 v_0} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left(\frac{\partial u_0}{\partial y}\right)_{y=0} = -[m_1\phi_2 + m_2\phi_3 + m_3\gamma_3] \tag{25}$$

4.0 Discussion and Results

The problem of MHD free convection dissipative flow with heat source and chemical reaction has been formulated and solved. To gain more insight into the study, the effects of the Grashof number,Gr, Magnetic field parameter, M, modified Grashof number, Gm, Porosity, K, chemical reaction, Kr, Schmidt number, Sc, Soret parameter, So, heat generation parameter, $\emptyset$, are studied numerically by using realistic values and the results displayed in graphical form. Figure 1 shows the effect of heat absorption on the velocity, it is observed that increase in the heat absorption, leads to general increase in velocity. The increase depends on the strength of the heat.

The influence of the Grashof number on the velocity is illustrated in figure 2. It is observed that increase in Grashof number contributes to the increase in velocity. The boundary layer thickness is increased by the thermal buoyancy force, this in effect enhances the fluid flow.

Figure 3 shows the effect of the magnetic parameter on the velocity. It is observed that the magnetic parameter decreases fluid flow when it is increased. This is mainly due to Lorentz force which condenses the momentum boundary thereby reducing the flow. The retarding effect is more pronounced at higher values of the magnetic field. It is shown in figure 4 that the porosity of the medium increases the velocity. The effect is very significant close to the plate.

The influence of the chemical reaction parameter is shown in figure 5. The profile shows that the chemical parameter enhances the flow of the fluid. This effect is pronounced as little increase leads to significant increase in the velocity.

Figure 6 shows the effect of the Schmidt number in the velocity. The indication on the profile is that increase in the Schmidt number decreases significantly the flow of the fluid. Physically increasing the Schmidt number increases the hydrodynamic layer hence reduced the mass transfer. It is shown in figure 7 that velocity increases with increase in the Soret parameter. Physically the observation is true, since increasing the Soret parameter implies increasing the driving force for the mass diffusion. The contribution of the heat source on the temperature of the fluid is shown in figure 8. It is observed that the heat source increases the temperature of the fluid. It adds to the already existing heat in the system. Figure 9 shows that increase in the oscillating parameter drops the temperature of the fluid. The effect is more pronounced far from the plate. The influence of the heat source is shown in figure 10. It is noticed that increase in the heat source increases the concentration of the fluid. Figure 11 shows the influence of Schmidt number on the concentration profile. It is observed that increase in the Schmidt number decreases the concentration of the fluid far from the plate. The concentration of the fluid decreases to near zero. The influence of the chemical reaction parameter on the concentration is shown in figure 12. The profile shows that increase in the chemical reaction decreases the concentration. Physically this is possible because chemical process reduces concentration at the boundary layer.

Figure 13 shows the effect of Soret on concentration. The profile shows that increase Soret appreciably increases the concentration of the fluid. The plate despite the influence far from the plate thermodiffusion is decreased and the concentration approaches a constant value near zero. Figure 14 shows the effect of heat source on the skin friction. It is noticed that the skin friction increases with simultaneous increase in the heat source and Grashof number. This relationship is seen to be of the linear type. The influence of the Grashof number on the skin friction is shown in figure 15. Increase in the Grashof number increases the skin friction. The profile indicates that far from the plate the effect is unchanged even with increase in the magnetic field. The influence of Soret on the skin friction is shown in figure 16. It reveals that skin friction increases with increase in Soret. The increase in skin friction is further enhanced by increase in porosity.
The effect of the heat source on heat transfer is shown in figure 17. It reveals that increase in heat source increases the heat transfer. The profile does reveals further that the variation does not affect the heat transfer in this case.

The influence of frequency of oscillatory parameter on heat transfer is shown in figure 18. The profile shows that increase in oscillation increases the rate of heat transfer. Simultaneous increase in the heat source shows further increase in the skin friction but much more pronounced at lower values of the oscillation. Figure 19 shows that increase in heat source enhances mass transfer. At lower values of the Schmidt number, this effect is minimal but becomes more pronounced at higher values even as the relationship is maintained. Figure 20 shows that increase in the chemical reaction increases the rate of mass transfer. At lower values of Soret a decrease is observed in the rate of mass transfer but increases afterwards and becomes equal for all values of the chemical reaction.

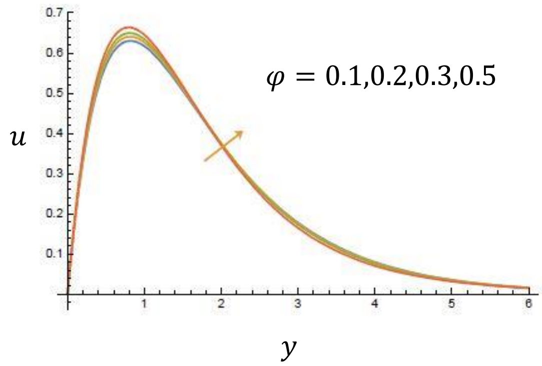

Figure 1: The effect of heat source on velocity for Sc=0.22, So= 0.5, ω=1, kr= 2, M= 0.5 k=0.1, Gr=5, Gm=5, t=0.

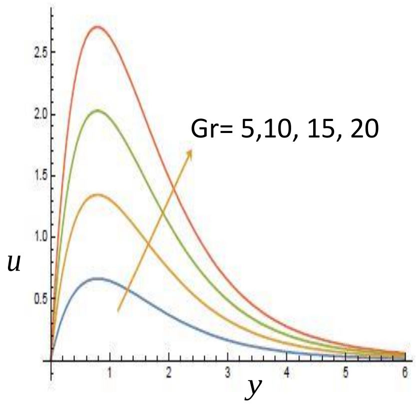

Figure 2: The effect of Grashof number on velocity for Sc=0.22, So= 0.5, ω=1, kr= 2, M= 0.5,k=0.1,Gm=5, φ=1.5, t=0

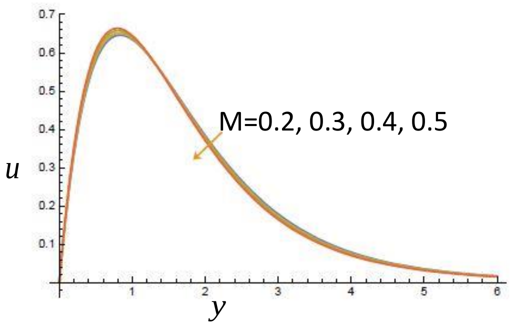

Figure 3: The effect of Magnetic Field on velocity for Sc=0.22, So= 0.5, ω=1, kr= 2, Gm= 5, k=0.1,Gr=5, φ=1.5, t=0

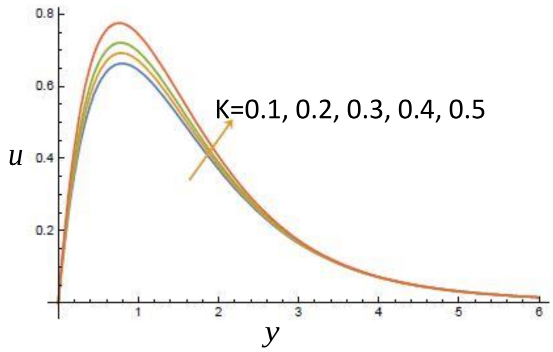

Figure 4: The effect of porosity on velocity for Sc=0.22, So= 0.5, ω=1, kr= 2, M=0.5, Gm= 5, Gr=5, t=0

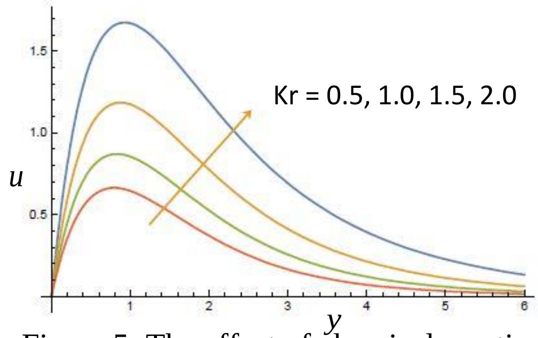

Figure 5: The effect of chemical reaction on velocity for Sc=0.22, So= 0.5, ω=1, k= 0.1, M=0.5, Gm= 5,Gr=5, φ=1.5,t=0.

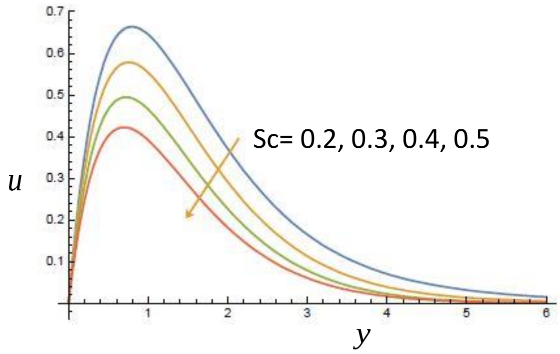

Figure 6: The effect of Schmidt number on velocity for, So= 0.5, ω=1, Kr=2, k= 0.1, M=0.5, Gm= 5,
Gr=5, φ=1.5

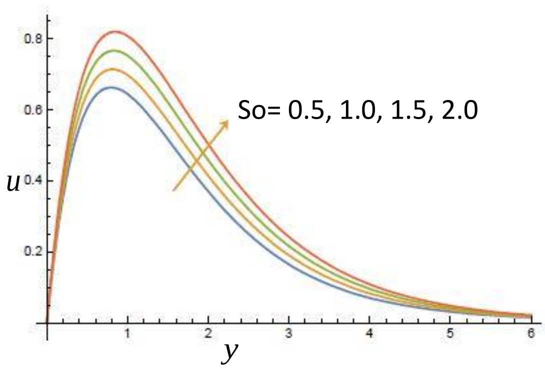

Figure 7: The effect of Soret on velocity for, Sc= 0.22, ω=1,k= 0.1, Kr=2, k= 0.1, M=0.5, Gm= 5, Gr=5, φ=1.5

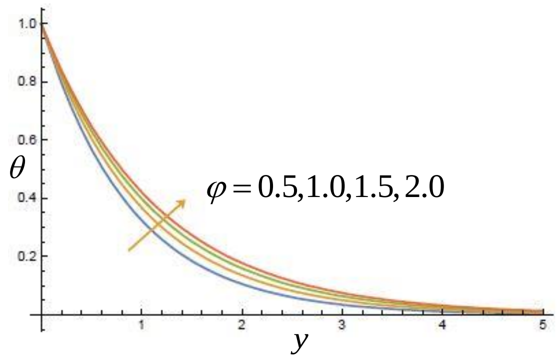

Figure 8: The effect of heat source on temperature for So= 0.5, Sc=0.22,Kr=2, k= 0.1, M=0.5, Gm= 5, Gr=5, ω=1, t=0

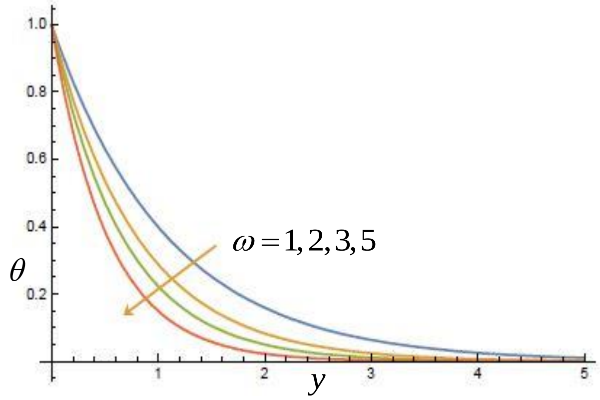

Figure9: The effect of oscillatory frequency on temperature for So= 0.5, Sc=0.22, Kr=2, k= 0.1, ϕ=1.5, M=0.5, Gm= 5,Gr=5, t=0

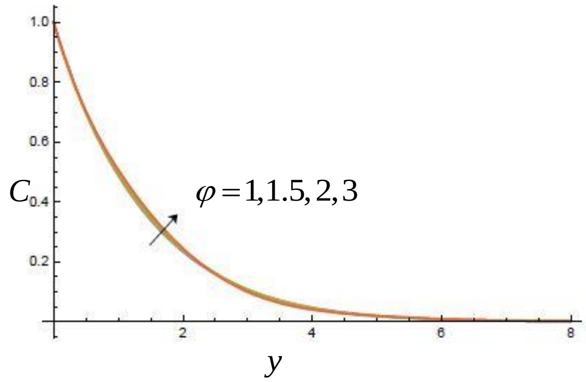

Figure10: The effect of oscillatory frequency on concentration for So= 0.5, Sc=0.22,Kr=2, k= 0.1, M=0.5, ω=1,Gm= 5,Gr=5, t=0

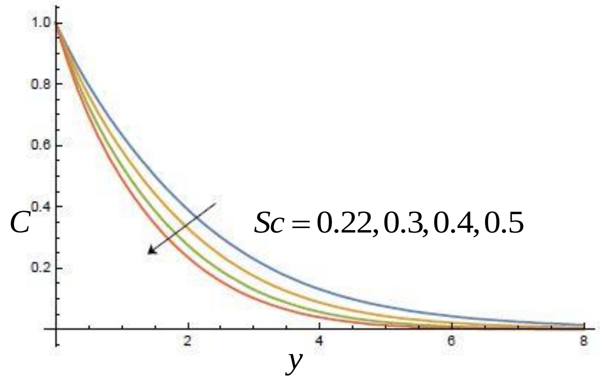

Figure11: The effect of Schmidt number on concentration for ϕ=1.5, ω=1,So=0.5, Kr=2, k= 0.1, M=0.5, Gm= 5,Gr=5, t=0

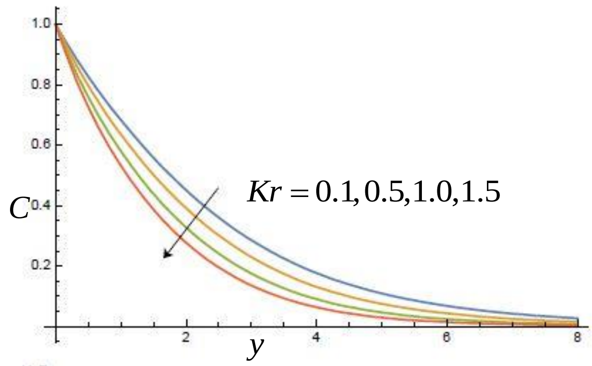

Figure12: The effect of chemical reaction on concentration for Sc=0.22, ϕ=1.5, So=0.5, ω=1, k= 0.1, M=0.5, Gm= 5,Gr=5, t=0

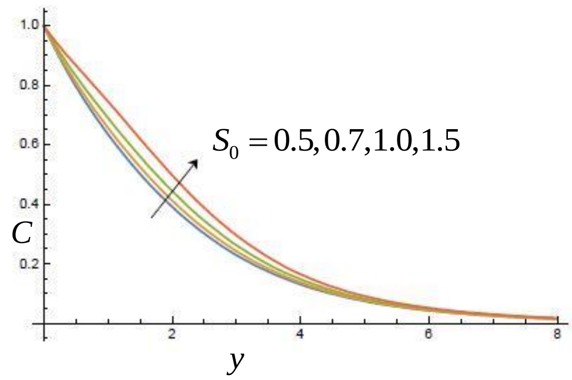

Figure13: The effect of Soret on concentration for Sc=0.22, ϕ=1.5 So=0.5,Kr=2, k= 0.1, ω=1, M=0.5, Gm= 5,Gr=5, t=0

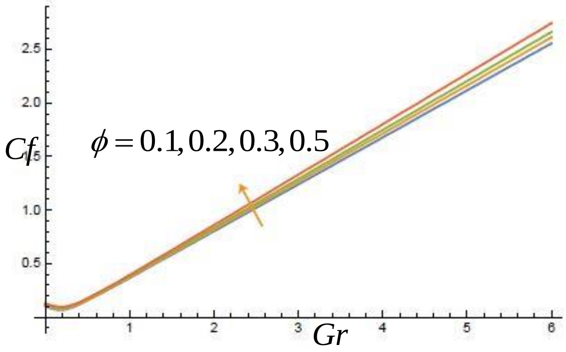

Figure14: The effect of heat source on rate of skin friction against Grashof number for Sc=0.22, So=0.5,Kr=2, ω=1, k= 0.1, M=0.5, Gm= 5, t=0

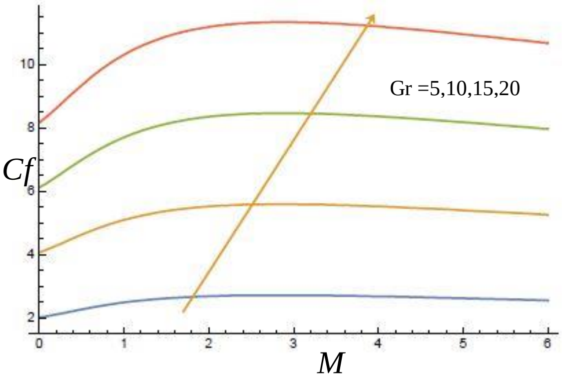

Figure15: The effect of Grashof number on rate of skin friction against magnetic field for Sc=0.22, φ=1.5 So=0.5, ω=1,Kr=2, k= 0.1, Gm= 5, t=0

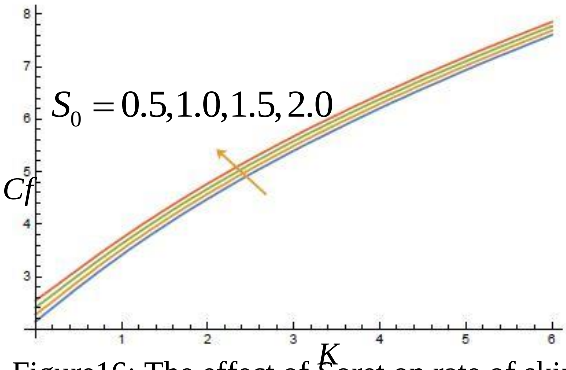

Figure16: The effect of Soret on rate of skin Friction against porosity parameter for Sc=0.22, ϕ =1.5, So=0 ω =1, k= 0.1, M=0.5, Gm= 5,Gr=5, t=0

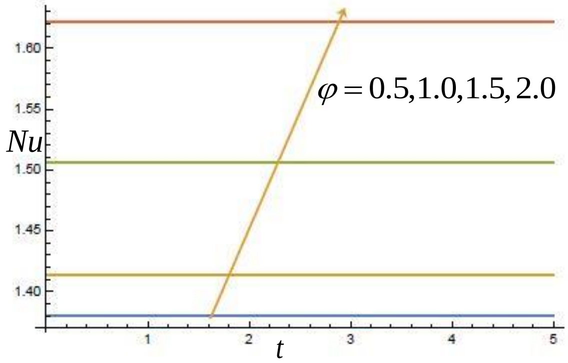

Figure17: The effect of heat source on rate of heat transfer against time for Sc=0.22,
So=0.5, ω =1,Kr=2, k= 0.1, M=0.5, Gm= 5, Gr=5

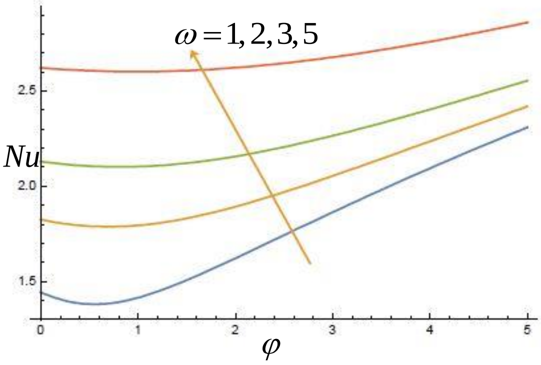

Figure 18: The effect of Oscillatory frequency on rate of heat transfer against heat source for Sc=0.22, So=0.5, φ =1.5, ω =1, Kr=2, k= 0.1, M=0.5,Gr=5, Gm= 5, t=0

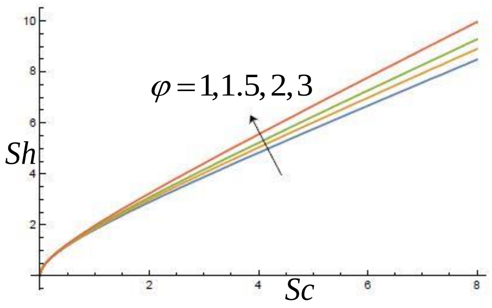

Figure 19: The effect of Oscillatory frequency on rate of shear stress against Schmidt number for Sc=0.22, So=0.5, ω=1,Kr=2, k= 0.1, M=0.5, Gm= 5,Gr=5, t=0

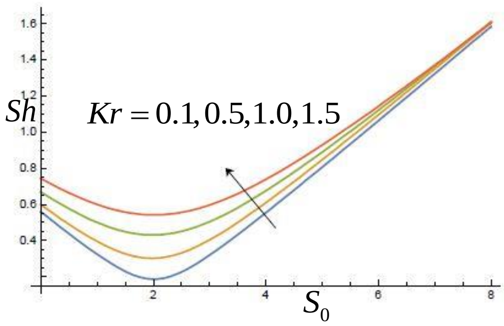

Figure 20: The effect of chemical reaction on rate of shear stress against Soret for Sc=0.22, ω=1, ϕ=1.5, k= 0.1, M=0.5, Gm= 5, Gr=5, t=0

5.0 Conclusion

A theoretical analysis is investigated for the problem of MHD free convective dissipative flow with heat source and chemical reaction, the following conclusion are drawn from our findings:

- **i.** Velocity increases with increase in heat absorption, chemical reaction and Grashof number, whereas there is a decrease with increase in Schmidt number and magnetic field.
- **ii.** Increase in oscillatory parameter decreases temperature and increases with an increase in heat source.
- **iii.** Concentration increases with increase in heat source and Soret but decreases with increase chemical reaction and Schmidt number.
- **iv.** Heat and mass transfer rates increase as the heat source increases.

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